Homework 2 Solutions

1. Problem 1, p. 131.

   (a) Give a truth table for $(s \vee t) \wedge (\neg s \vee t) \vee (s \vee \neg t)$

s	t	$\neg s$	$\neg t$	$s \vee t$	$\neg s \vee t$	$s \vee \neg t$	$(s \vee t) \wedge (\neg s \vee t) \vee (s \vee \neg t)$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	F	T	T	F	F
F	F	T	T	F	T	T	F

   (b) Give a truth table for $(s \implies t) \wedge (t \implies u)$

s	t	u	$s \implies t$	$t \implies u$	$(s \implies t) \wedge (t \implies u)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

2. Problem 4, p. 131.

s	t	$\neg s$	$s \implies t$	$\neg s \vee t$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

   Since the two rightmost columns are identical, the two statements are equivalent.

3. Problem 6, p. 131.

p	q	$p \oplus q$	$p \wedge \neg q$	$\neg p \wedge q$	$(p \wedge \neg q) \vee (\neg p \wedge q)$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

Since column two and column five identical, the two statements are equivalent.

4. Problem 2, p. 147.

This is true, because we just need to show that **one integer** satisfies the equation. For example, let $x = 3$. Then:

$$\begin{aligned} (x-2)^2 + 1 &\leq 2 \\ (3-2)^2 + 1 &\leq 2 \\ 1^2 + 1 &\leq 2 \\ 2 &\leq 2 \end{aligned}$$

5. Problem 7, p. 147.

(a) False. Because this assertion is "for all positive integers," only one counterexample is necessary to disprove the statement. Let $z = 1$. Then:

$$\begin{aligned} z^2 + 6z + 10 &> 20 \\ 1^2 + 6 \times 1 + 10 &> 20 \\ 1 + 6 + 10 &> 20 \\ 17 &\not> 20 \end{aligned}$$