Homework 2 Solutions

1. Problem 1, p. 131.

   (a) Give a truth table for $(s \vee t) \wedge (\neg s \vee t) \vee (s \vee \neg t)$

s	t	$\neg s$	$\neg t$	$s \vee t$	$\neg s \vee t$	$s \vee \neg t$	$(s \vee t) \wedge (\neg s \vee t) \vee (s \vee \neg t)$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	F	T	T	F	F
F	F	T	T	F	T	T	F

   (b) Give a truth table for $(s \implies t) \wedge (t \implies u)$

s	t	u	$s \implies t$	$t \implies u$	$(s \implies t) \wedge (t \implies u)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

2. Problem 4, p. 131.

s	t	$\neg s$	$s \implies t$	$\neg s \vee t$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

   Since the two rightmost columns are identical, the two statements are equivalent.

3. Problem 6, p. 131.

p	q	$p \oplus q$	$p \wedge \neg q$	$\neg p \wedge q$	$(p \wedge \neg q) \vee (\neg p \wedge q)$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

   Since column two and column five identical, the two statements are equivalent.

4. Problem 2, p. 147.

   This is true, because we just need to show that **one integer** satisfies the equation. For example, let $x = 3$. Then:

$$\begin{aligned} (x-2)^2+1 &\leq 2 \\ (3-2)^2+1 &\leq 2 \\ 1^2+1 &\leq 2 \\ 2 &\leq 2 \quad QED \end{aligned}$$

5. Problem 7, p. 147.

   (a) False. Because this assertion is "for all positive integers," only one counterexample is necessary to disprove the statement. Let $z = 1$. Then:

$$\begin{aligned} z^2+6z+10 &> 20 \\ 1^2+6\times 1+10 &> 20 \\ 1+6+10 &> 20 \\ 17 &\not> 20 \quad QED \end{aligned}$$

   (b) True. This is trickier because we have "for all," which means the statement must work for every valid integer. However:

   - The smallest $z^2 - z$ can be is 0 when $z = 0$ ($0^2 - 0 \geq 0$ is true).
   - Any negative value for z makes the computation larger because the square will be positive and the subtraction turns into addition. For example, if $z = -1$, then $(-1)^2 - (-1) = 2 \geq 0$.
   - Any positive integer squared will be itself or greater, which means the result will be greater than 0.

   (c) False. Looking at the previous problem, any positive integer squared will be itself or greater. This means that, in general, that when the square is subtracted from the number itself, the result will be negative which is not greater than 0. The smallest z can be is 1, where $1 - 1 \not> 0$. Thus, there does not exist any positive integer that makes the statement true.

(d) True. We need to find just one integer that works. In this case, let $z = 3$. Then:

$$\begin{aligned} z^2 - z &= 6 \\ 3^2 - 3 &= 6 \\ 9 - 3 &= 6 \\ 6 &= 6 \quad QED \end{aligned}$$